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NEW TECHNOLOGY SPILLOVERS INTO THE PAYMENT SYSTEM*

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In modern economies, multiple means of payment associated with the exchange of goods coexist. This paper examines one such payment system in an economy with endogenous technological change. It consists of money and a costly accounting system that receives spillovers from new technologies. Positive nominal interest rates are shown to produce welfare losses by inducing a reallocation of human capital into the payment system, and out of the production of final goods and new knowledge. The former substitution produces level effects on output and the latter produces growth effects. At higher levels of inflation, these marginal effects are seen to be weaker.

In modern economies, multiple means of payment associated with the exchange of goods coexist. Among the available options that facilitate these transactions are costly accounting systems such as credit and debit cards for households and cash concentration and sweep accounts for firms. These systems ultimately depend upon various methods of implementing electronic funds transfers (EFTs), whose efficiency in turn is dependent upon computer and information systems technology. Therefore, the efficacy of these means of payment in minimising fundamental trading frictions can be enhanced by improvements in the techniques for implementing EFTs, or by the 'spillovers' of advances in computer and information systems technology via their application to the payment system. This technology interacts with monetary policy in determining the structure of the economy's payment system. In particular, as inflation rises, agents respond by selecting a payment system with a mix of media of exchange that is more heavily weighted in favour of costly accounting systems to avoid the inflation taxes imposed on cash transactions.[1] This may misallocate resources and thus reduce welfare.

Marquis and Reffett (1992*b*) examine one such payment system in a stochastic neoclassical growth model where the costs of operating the accounting system arise from using physical capital to run the system. In this economy, inflation taxes are shown to produce level effects on output even when a cash-in-advance constraint is imposed only on a subset of the consumption goods. This contrasts with Abel's (1985) result for a deterministic, cash-in-advance economy, where it is shown that the cash-in-advance constraint must also be imposed on investment goods for anticipated inflation

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[1] Prescott (1987), Marquis and Reffett (1992*b*), Lacker and Schreft (1991), Schreft (1992*a*, *b*), Ireland (1994*a*, *b*), and Ireland and Dotsey (1993) explicitly model the costs associated with private efforts to minimise the incidence of an inflation tax by varying the mix of exchange media used.

taxes to affect the steady-state level of output.[2] In the more sophisticated payment system of Marquis and Reffett (1992*b*), households alter the structure of the payment system as an endogenous response to inflation taxes.[3] This response involves capital reallocations away from (toward) the production of goods and toward (away from) the production of transaction services in response to a positive (negative) inflation shock. Financial innovations in the form of shocks to the technology for transforming the existing computer/information systems technology into transaction services were also shown to induce capital reallocations. In this case, the accounting system is more intensively utilised, implying less reliance on cash; while some portion of the economy's capital stock formerly used to produce transactions is freed for use in the production of goods.[4]

This paper extends the Marquis and Reffett (1992*b*) model to include endogenous growth and thus examine how inflation taxes alter growth rates by inducing an endogenous change in the structure of the payments system.[5] The payment system consists of money and a costly accounting system that requires human capital as an input into the production of transaction services. Endogenous growth is generated by a Romer (1990) model of 'endogenous technological change'. This technological advance produces spillovers into the payment system by raising the marginal product of human capital in that activity. Human capital has alternative uses in the production of final goods and in the production of new knowledge in the research sector. Positive nominal interest rates thereby induce reallocations of human capital into the production of transaction services, and out of the production of final goods and new knowledge. The former substitution produces level effects on output; the latter growth effects. At higher levels of inflation, these marginal effects of the reallocation of human capital are shown to be weaker, because the structure of the payment system has been altered until the volume of real cash transactions as a percentage of total consumption is relatively small, so that the incremental inflation tax is less distorting of the agent's optimal consumption bundle.

The paper is organised as follows. The various sectors of the model economy are described in Section I. In Section II, the equilibrium allocations of human capital associated with a specific family of (implementable) equilibrium balanced growth paths are described. These equilibria are indexed by the nominal interest rate, which serves as a measure of the monetary distortion resulting from the selection of a monetary rule. Members of this family of

[2] For other papers that examine the effects of inflation taxes in cash-in-advance models, see Cooley and Hansen (1989, 1991), DeGregorio (1991, 1992), Stockman (1981), Coleman (1990), Fuerst (1992), and Marquis and Reffett (1992*a*).

[3] Engineer and Berhardt (1991) demonstrate how a sufficiently high inflation tax can fundamentally alter the structure of a payment system whereby a monetary equilibrium collapses, and the economy resorts to barter.

[4] This freeing of real resources for uses other than to conduct transactions (for example, to produce goods) is emphasised by Black (1970).

[5] Ireland (1994*a*) also examines an elaborate payment system in an endogenous growth model that allows households to select a costly credit arrangement (as in Schreft (1992)) as an alternative to cash. However, he focuses on the positive predictions of his model for the behaviour of velocity, rather than on the growth effects of inflation.

equilibria are examined to determine how changes in the nominal interest rate induce both level and growth effects in the real sectors of the economy. In Section III, the monetary rules that support these nominal interest rates are identified. The distortion is then rewritten in terms of inflation, which is used to index the equilibria. The economy's sensitivity to incremental inflation taxes as the inflation tax increases is then evaluated. A numerical example of the model is provided in Section IV to illustrate the theoretical predictions. Section V summarises the paper's conclusions.

I. A MODEL ECONOMY: SECTOR BY SECTOR

The mechanism for endogenous growth in this model is nearly identical to Romer (1990). New knowledge is a non-rival good that is produced in the research sector and protected by patents. Each patent prevents use of the particular assemblage of new knowledge which is codified in the design of a unique intermediate good. This renders new knowledge partially excludable, so production of the intermediate good generates monopoly profits, which in turn determine the patent's price. However, alternative uses of this new knowledge are not excludable, and its non-rival character produces spillover effects that can enhance other economic activities. One such activity is the process of research itself, since new knowledge builds on old knowledge. It is this 'positive feedback' in the knowledge creation process that generates growth endogenously. This mechanism also produces non-Pareto optimal allocations of resources due to the monopoly profits from the production of intermediate goods and the partial non-excludability of new knowledge. In this model, a transaction services sector exists that also receives spillovers from the research sector, through enhanced productivity of human capital in the provision of transaction services.

Money enters into the model via a cash-in-advance constraint that is imposed on a subset of the consumption goods purchased by the (representative) household. Inflation taxes alter resource allocations by influencing the selection of the economy's payment system. The solution procedure is carried out via profit maximisation in the research, intermediate goods, final goods, and transaction services sectors in real terms, as if the economy were non-monetary. Alternative monetary rules that produce positive nominal interest rates then only affect the household's decisions. The optimality conditions for each of these sectors are described below.

(a) *Final Goods Sector*

Output in the final goods sector, y_t, is given by a convex production technology that is homogenous of degree one in its factor inputs that consist of human capital, h_{yt}, and a vector of intermediate goods, x_{it}, $i = 1, \ldots, \infty$. Throughout the paper, all quantities are measured in per capita terms. This technology is:

$$y_t = h_{yt}^{\alpha} \sum_{i=1}^{\infty} x_{it}^{(1-\alpha)} \quad \alpha \in (0, 1). \tag{1}$$

Each of the intermediate goods is produced in accordance with a unique patented design that is developed in the research sector. The number of these designs is determined by the cumulative amount of research effort over time. At a point in time, the number of patents is given by A_t such that $x_{it} > 0$, $\forall i \leqslant A_t$ and $x_{it} = 0$, $\forall i > A_t$. Perfectly competitive final goods producers rent human capital from households at the real rental rate of w_{yt}. Intermediate goods are infinitely durable (with no depreciation) and are rented from the intermediate goods producer at the real rental rate of q_{it}.

(b) Intermediate Goods Sector

The allocation of final goods between consumption and investment goods is made by the representative household. Investment goods add to the physical capital stock, k_t, which is owned by the household and rented to the intermediate goods sector as the sole factor of production. The real rental rate on physical capital is r_t. Assuming that η units of foregone consumption are required to produce one unit of any of the intermediate goods, the total physical capital stock, k_t, is related to the stock of intermediate goods by the following expression.

$$k_t = \eta \sum_{i=1}^{A_t} x_{it}, \quad \eta \in (0, \infty). \tag{2}$$

Each producer of intermediate goods purchases an infinitely-lived patent on the design of its goods from the research sector. As the sole producer of the good, it is able to extract the monopoly profits from sales of the good, Π_{xit}. It therefore faces the downward-sloping demand curve from the final goods producer, and pays rent to households on the portion of the economy's physical capital stock, k_{it}, that it employs in the production of x_{it}.

Potential intermediate goods producers compete for the monopoly rights associated with the patent. The real market price of the newly-created patent, P_t, on the design of x_{it} is given by the present value of the monopoly profits that it generates. Since the intermediate goods producer maximises profits each period, the infinitely lived patent sold at date t will be priced as a consol, with the discount rate equal to the real rental rate on capital, r_t.

$$P_t = \sum_{t=0}^{\infty} \Pi_{xit} / \Pi_{s=0}^{t} (1 + r_s). \tag{3}$$

(c) Research Sector

The research sector consists of a large number of identical profit maximising research firms. These firms produce the non-rivalrous good 'new knowledge'. However, a particular assemblage of this 'new knowledge' is embodied in new patentable designs for intermediate goods. The level of knowledge is thus measured as A_t, and places a limit on the number of existing designs. The technology for creating new knowledge is assumed to be intensive in its use of human capital, h_{At}, which is taken as the sole factor input. However, given that the creation of new knowledge is assumed to build on the existing level of knowledge, then subject to the patents' rights, 'new knowledge' produces spillovers into the research process. These spillovers create the engine of growth for the economy. This is captured by a linear production technology for the research sector.

$$A_{t+1} = (1+\delta h_{At})\, A_t, \quad \delta \in (0, \infty). \tag{4}$$

Competition in the human capital market exists among research firms, who pay households a competitive real rental rate, w_{At}. The magnitude of w_{At} reflects the revenues from patent sales to intermediate goods producers. Therefore, in the absence of patent protection, research activity would cease, and as shown below, the economy would stagnate.

(*d*) *Transaction Services Sector*

The representative household purchases a subset of its consumption goods by using a transaction services facility instead of cash. This facility is costly to operate. A large number of identical competitive transaction services firms possess technologies that require human capital, h_{zt}, to produce transaction services. However, much of the knowledge created produces spillovers into the production technology for transaction services by improving the efficiency of the information systems upon which it relies. The marginal product of human capital in the production of transaction services thereby increases with the existing level of knowledge, A_t. The production function for the transaction services firm is given by:

$$z_t = \gamma A_t h_{zt}, \quad \gamma \in (0, \infty), \tag{5}$$

where z_t is the volume of real transactions produced.

Transaction services firms sell these transactions to the households at the real unit price of d_t. The zero profit condition for these firms determines a competitive real rental rate of w_{zt} paid to households from whom human capital is rented.

(*e*) *Household Sector*

Households receive all output from the final goods sector and own the economy's resources. Their optimisation problems are used to characterise the supply schedules for human and physical capital, and the demand schedules for final goods, transaction services, and money. The representative household maximises lifetime utility subject to a feasible set of choices described by the technology and resource constraints given below. Lifetime utility is defined by:

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}), \quad \beta \in (0, 1) \tag{6}$$

where $U(c_{1t}, c_{2t}) = \ln c_{1t} + a \ln c_{2t}$, $a \in (0, \infty)$. Therefore, this paper restricts attention to preferences characterised by equation (6).

The consumption goods differ only by the medium of exchange needed to acquire them.[6] The 'cash good', c_{1t}, is subject to the following cash-in-advance constraint:

$$c_{1t} \leqslant (M_t^d + J_t)/p_t, \tag{7}$$

[6] We have in mind a spatial trading environment similar to Marquis and Reffett (1992*b*) which would endogenously support the payment system constraints on the household's programming problem.

where M_t^d is the stock of money carried over from the previous period, J_t is the lump-sum, per capita monetary transfer that the household receives at the beginning of the period, and p_t is the money price of output. The second good, which is referred to as the 'non-cash good', c_{2t}, is subject to a capacity constraint imposed by the transaction services facility, where each unit of transaction services purchases the same volume of the non-cash good.[7]

$$c_{2t} \leqslant d_t z_t. \tag{8}$$

From equation (5), it is evident that this capacity constraint can be relaxed directly by increasing the allocation of human capital to the transaction services sector, or indirectly by receiving a greater spillover from the research sector due to a higher level of knowledge creation through research.

The representative household must allocate its stock of human capital, h, between the three competing uses in the production of final goods, new knowledge, and transaction services. Thus,

$$h_{yt} + h_{At} + h_{zt} \leqslant h. \tag{9}$$

The budget constraint that is faced by the household consists of rental income from human and physical capital and a post transfer stock of real balances that constrains the purchases of consumption goods, investment goods, and transaction services, while carrying forward a stock of money to meet next period's cash requirements.

$$c_{1t} + c_{2t} + (k_{t+1} - k_t) + d_t z_t + M_{t+1}^d / p_t \leqslant w_{yt} h_{yt} + w_{At} h_{At} + w_{zt} h_{zt} + r_t k_t + (M_t^d + J_t)/p_t. \tag{10}$$

The household's decisions are characterised by the solution to a dynamic optimisation problem expressed as the maximisation of lifetime utility, (6), given the initial (household) state (h, k_0, M_0), by choosing the vector of optimal sequences for the consumption bundle, next period's physical capital stock (or investment), the allocation of human capital, the volume of transaction services to purchase, and next period's initial nominal money holdings (that is $\{c_{1t}, c_{2t}, k_{t+1}, h_{yt}, h_{At}, h_{zt}, z_t, M_{t+1}^d\}_{t=0}^{\infty}$), from among the feasible set defined by equations (7)–(10) and by the non-negativity constraints on each of the decision variables.

The household optimality conditions for this monetary economy can be summarised by a set of equations (11)–(14), with the following economic interpretations. The household chooses the allocation of its stock of human capital to equate marginal returns across sectors. This equalises the sector-specific real rental rates.

$$w_{yt} = w_{At} = w_{zt} = w_t. \tag{11}$$

The effect on household decisions of positive nominal interest rates, denoted R_t and defined as in Townsend (1987) and Lucas and Stokey (1987),[8] is to

[7] Inflation tax distortions in this model cause the household to alter the composition of its consumption bundle toward the 'non-cash' good and away from the 'cash' good, which reduces welfare.

[8] Define λ_{1t} and λ_{2t} as the Lagrange multipliers on the budget and cash-in-advance constraints. Then, the nominal interest rate is defined as $1 + R_t = (\lambda_{1t} + \lambda_{2t})/\lambda_{1t}$. Note that relaxation of the cash-in-advance constraint each period, or $\lambda_{2t} = 0$, $\forall t \geqslant 0$ corresponds to a zero nominal interest rate.

distort the temporal consumption bundle effectively away from consumption of the cash good, c_{1t}, and toward consumption of the non-cash good, c_{2t}, according to equation (12).

$$c_{1t} = [2/a(1+R_t)]\, c_{2t}. \tag{12}$$

In addition, the intertemporal marginal rate of substitution for the cash good is altered, as given by:

$$c_{1t+1}/c_{1t} = \beta(1+R_t)/\pi_{t+1}, \tag{13}$$

where the inflation factor is defined as $\pi_{t+1} = p_{t+1}/p_t$. By contrast, the intertemporal marginal rate of substitution in consumption of the non-cash good, c_{2t}, is determined by the opportunity costs of foregone consumption of c_{2t} in the current period. This is the discounted, single period gross return to physical capital:

$$c_{2t+1}/c_{2t} = \beta(1+r_{t+1}). \tag{14}$$

Therefore, positive nominal interest rates only affect this optimality condition indirectly via equilibrium conditions.

(f) Monetary Authorities

The focus of this paper is on the welfare costs of inflation due to the misallocation of resources via the payment system. The role of government is therefore limited to supplying money uniformly to all households according to the following state-dependent rule:

$$M_{t+1} = G(\mathbf{S}_{t+1})\, M_t, \tag{15}$$

where M_t is the aggregate per capita money supply, $\mathbf{S}_t$, is the vector of aggregate state variables (h, k_t, A_t) and $G(\mathbf{S}_t)$ is the gross growth rate of the money supply. The lump-sum monetary transfers, J_t, can be determined by equation (15), such that in equilibrium: $M_t = M_t^d + J_t$.

II. POSITIVE NOMINAL INTEREST RATES AND HUMAN CAPITAL ALLOCATIONS

Positive nominal interest rates arise from inflation taxes on cash purchases of consumption goods. In response to these taxes, households substitute away from the consumption of cash goods and toward the consumption of non-cash goods. This induces an endogenous selection of a payment system that reflects the greater share of non-cash transactions. However, these transactions are costly to produce and require a reallocation of the economy's human capital resources. This reallocation produces 'level effects' on output, factor intensities, and rental rates. It also alters the minimum stock of human capital that is required for the economy to avoid stagnation. When the stock of human capital exceeds this minimum, this reallocation affects the growth rates of consumption, output, the capital stock, and 'new knowledge'. These results are summarised below in Propositions 1 to 4.

To derive these results, it is first necessary to exploit the symmetry imposed on the intermediate goods producers as in Romer (1990). For a uniform (or economy-wide) rental rate for physical capital, r_t, all firms in the intermediate goods sector choose to produce in each period according to an identical monopoly solution, such that $x_{it} = x_t$ and $q_{it} = q_t$, $\forall i < A_t$. This results from similar production processes for these firms in transforming physical capital into intermediate goods, and from the fact that they face identical derived demand schedules for their products from the final goods producers. This further implies that all of these firms receive the same monopoly profits each period, or $\Pi_{it} = \Pi_t$, $\forall i \leqslant A_t$. With this information, the dynamic solution to the model for the particular family of equilibrium balanced growth paths of interest can be found.

However, to analyse the effects of the inflation tax, it is necessary to identify equilibrium balanced growth paths where the distortionary effects of inflation on resource allocations are held constant over time. The solution that follows is for one such family of (implementable) equilibria, where each member of that family is associated with a constant nominal interest rate, R, which can therefore be used to index the equilibria. The unique monetary rule that implements a given distortion, R, is subsequently identified. Each of these equilibria is further characterised by constant allocations of human capital, h_y, h_A, and h_z, out of the total available stock, h, a constant real rental rate on physical capital, r, and a constant patent price, P.[9]

Solving the model for these equilibrium paths results in the following human capital allocations, given a nominal interest rate, R:[10]

$$h_y = \{\psi h - [(1-\psi)(1-\beta)]/\delta\}\Delta, \tag{16}$$

$$h_A = -[(1-\beta)/\delta] + \beta(1-\alpha)\, h_y, \tag{17}$$

$$h_z = h + [(1-\beta)/\delta] - [1+\beta(1-\alpha)]\, h_y, \tag{18}$$

where $\Delta \equiv \{[1/(1-\alpha)] - \beta(1-\alpha) + \psi[1+\beta(1-\alpha)]\}$, with the monetary distortion isolated in $\psi \equiv \{1+[2/a(1+R)]\}[\alpha/(1-\alpha)]$, and where these allocations are subject to the constraints h_y, h_A, $h_z \in [0, h]$. A reasonable parameter value for the factor share on human capital in the production function for the final goods sector, i.e., $\alpha \in (0{\cdot}5, 1)$, is sufficient (but not necessary) to show that $h_y > 0$, $\forall h > 0$.[11] The equilibrium rental rate on physical capital and the equilibrium patent price are given by:

$$r = (1-\alpha)\,\delta h_y, \tag{19}$$

$$P = (\alpha/\delta)(x/h_y)^{(1-\alpha)} \tag{20}$$

where the equilibrium quantity of intermediate goods, x, is given by:

$$x = [(1-\alpha)\, h_y^{(\alpha-1)}/\delta\eta]^{(1/\alpha)}. \tag{21}$$

Along these paths, consumption of both goods, c_{1t} and c_{2t}, output, y_t, the capital

[9] These latter requirements are essentially identical to those imposed by Romer (1990) in his selection of a 'balanced growth path' for study.

[10] The solution procedure, along with proofs of the subsequent propositions, is contained in an appendix that is available from the authors.

[11] The inequality that must hold for $h_y > 0$ is $h > (1-\psi)(1-\beta)/\delta\psi$, which is true for $\psi > 1$, which is satisfied for $\alpha > 0{\cdot}5$. Labour's share of factor income in the United States is close to two-thirds (see Kydland and Prescott (1982)).

stock, k_t, and the level of knowledge, or number of intermediate goods, A_t, are all growing at the same rate, given by the growth factor, θ.

$$\theta = \beta[1+(1-\alpha)\,\delta h_t] = (1+\delta h_A) \geqslant 1. \tag{22}$$

Having solved for this family of equilibrium balanced growth paths, the effects of higher inflation taxes, and thus higher nominal interest rates can be determined. The results are summarised in Propositions 1 to 4 below.

PROPOSITION 1 (human capital allocations): *An increase in the nominal interest rate, R, causes human capital to be allocated away from the production of final goods, $dh_y/dR < 0$, and away from research, $dh_A/dR < 0$, and toward the production of transaction services, $dh_z/dR < 0$.*

This reallocation of human capital is the household's response to the increased cost of buying the cash good due to the higher nominal interest rate (inflation tax). The household chooses to reduce cash good consumption, c_{1t}, and increase non-cash good consumption, c_{2t}. However, to acquire more of the non-cash good, production of transaction services, z_t, must increase. This requires a greater allocation of human capital to transaction services, h_z, and comes at the expense of human capital allocations to the production of final goods, h_y, and to research, h_z.

The reduction in the quantity of human capital used to produce final goods lowers output, and differentially affects the factor markets according to the following proposition.

PROPOSITION 2 (level effects): *An increase in the nominal interest rate, R, lowers output in the final goods sector, $dy_t/dR < 0$, provided the factor share on human capital is sufficiently high ($\alpha > 0{\cdot}5$), but with the production in this sector becoming relatively more intensive in its use of intermediate goods, $d(k/h_y)/dR > 0$. This is accompanied by a lower rental rate on physical capital, $dr/dR < 0$, and a higher rental rate on human capital, $dw_t/dR > 0$.*

This increase in physical capital intensity in the production of final goods is further enhanced by an increase in the production of intermediate goods. This raises the stock of physical capital, k.[12]

For the economy to experience positive growth, the household must allocate some portion of its human capital to research, i.e. h_A must be positive. The condition for growth can therefore be found from equations (16) and (17) to require the stock of human capital to be sufficiently high relative to a critical value, h_c:

$$h > h_c \equiv [(1-\beta)/\delta\psi]\{(1-\psi)+[\Delta/\beta(1-\alpha)]\}. \tag{23}$$

This result is similar to Romer (1990), and arises because human capital is used in research to expand future production prossibilities, and thus raise future consumption and utility. However, there is an opportunity cost incurred in terms of foregone output in the current period; this produces a marginal reduction in lifetime utility. Given that the future is discounted, the marginal product of human capital in the research sector must be sufficiently high to

[12] This 'enhancement' result would not generalise to a model where intermediate goods production was more human capital intensive than final goods production.

warrant this utility loss. Therefore, the higher is the quality of research, i.e. the higher is δ, the lower is this critical level of human capital needed to support growth. However, the minimum critical value required for growth is determined by the monetary distortion as measured by the nominal interest rate, R. This is written as the following proposition.

PROPOSITION 3 (non-stagnation condition): *The minimum value of human capital required for the economy to experience positive growth*, h_c, *is an increasing function of the nominal interest rate*, R, *or* $dh_c/dR > 0$.

As the nominal interest rate rises, more human capital is diverted into the production of the transaction services needed to acquire the non-cash good. Therefore, less human capital remains available for final goods production and research. This suggests that the nominal interest rate could be sufficiently high so as to completely eliminate growth.[13] Otherwise, increases in the nominal interest rate reduce growth rates according to the following proposition.

PROPOSITION 4 (growth effects): *An increase in the nominal interest rate*, R, *reduces the growth rate of consumption of both goods*, c_{1t} *and* c_{2t}, *of output*, y_t, *of the capital stock*, k_t, *and of knowledge*, A_t, *if the stock of human capital is sufficient to support growth. That is*, $d\theta/dR < 0$, *whenever* $h > h_c$.

A higher nominal interest rate lowers the stock of human capital devoted to research, h_A, which reduces the rate of expansion of new knowledge, A_t. Since the level of new knowledge ultimately determines the real rental rate on human capital, it also grows more slowly, implying slower productivity growth. The rate at which newly patented designs for intermediate goods are created is slowed, which reduces the expansion of intermediate goods, thereby retarding the growth rate of output in the final goods sector. As a consequence, both consumption and physical capital are growing more slowly.

Positive nominal interest rates (inflation taxes) therefore produce welfare losses associated with reductions in lifetime utility resulting from both the equilibrium level and growth effects of the distortion. The levels effects induce the selection of a suboptimal consumption bundle, with too much consumption of the non-cash good and too little consumption of the cash good. The growth effects lower the total volume of consumption in all future periods, which cumulatively generates the present value utility losses.[14]

III. MONETARY RULES AND INFLATION

The previous section solved for a continuum of balanced growth paths indexed by the nominal interest rate. To identify the monetary rules that support these equilibria, it is necessary to find the mapping from the gross money supply

[13] Marquis and Reffett (1991, 1992*c*) also obtain this result in endogenous growth models with cash-in-advance imposed on investment purchases of the human or physical capital good that in the absence of the monetary distortions generates growth.

[14] In the non-monetary version of this model examined by Romer (1990), the monopoly profits associated with the patent rights on 'new designs' and the partial non-excludability of 'new knowledge' both cause the first welfare theorem to break down. The competitive equilibrium allocation of human capital to research is less than optimal. As indicated by Proposition 1, positive nominal interest rates further reduce h_A, and therefore induce additional welfare losses.

growth rate, $G(\mathbf{S}_t)$, to the nominal interest rate, R_t. For a constant nominal interest rate, this monetary rule can be found from equations (7) and (13) and money market equilibrium, to consist of a constant growth rate, G.

$$G(\mathbf{S}_t) = G = \beta(1+R). \tag{24}$$

Within the second best context of this model, Friedman's Rule of a zero nominal interest rate requires the money supply to grow at a rate that is just sufficient to relax the cash-in-advance constraint each period (which implies that the multiplier on the cash-in-advance constraint $= 0$, $\forall t$).[15] In this case, the gross inflation rate, π^* (which corresponds to a net deflation) is given by:

$$\pi^* = \beta/\theta^*, \tag{25}$$

where θ^* is the gross growth rate of the economy when $G = \beta$. The value, θ^*, can be found from equations (17) and (22), with h_y determined from equation (16) and the nominal interest rate set to zero. This is the maximum possible value of θ. To determine whether the economy can achieve positive growth under any monetary rule, the stock of human capital, h, must exceed the critical value, h_c^*, which is the value of h_c obtained from equation (23) with the nominal interest rate set to zero.

If $h \leqslant h_c^*$, then the economy does not experience positive growth, $\theta^* = 1$, and the gross inflation rate under the Friedman rule is $\pi^* = \beta$, as in the standard neoclassical growth model. (See, e.g. Cooley and Hansen (1989).) However, when $h > h_c^*$, the economy experiences positive endogenous growth (in the absence of a monetary distortion), and the inflation rate is lower than β, as given by equation (23) when $\theta^* > 1$.

To determine the sensitivity to anticipated inflation of the equilibrium allocations of human capital, it is first necessary to find the mapping of the nominal interest rate into the gross inflation rate. Equations (13), (14), and (22) can be combined to yield the following expression:

$$(1+R) = [1+(1-\alpha)\,\delta h_y]\,\pi. \tag{26}$$

Using equation (26) to reindex the family of equilibrium balanced growth paths that are being examined, Propositions 1 to 4 can be rederived with the monetary distortion expressed as the gross inflation rate. In addition, the sensitivity of these results at various levels of inflation can be obtained, and summarised by the following proposition.[16]

PROPOSITION 5 (sensitivity to inflation taxes): *The effects of the inflation tax on equilibrium allocations of human capital, and therefore on the growth rate of the economy, are less pronounced at higher levels of inflation. That is,* $d^2h_y/d\pi^2 > 0$, $d^2h_A/d\pi^2 > 0$, $d^2h_z/d\pi^2 < 0$, *and* $d^2\theta/d\pi^2 > 0$.

These results indicate that the incremental level and growth effects of inflation taxes are mitigated at higher levels of inflation by the fact that the

[15] While this corresponds to the Friedman (1969) rule, optimal monetary policy does not restore Pareto optimal allocations in this model for the reasons described in footnote 14.

[16] Note that equation (26) introduces a nonlinearity in the set of equations that are used to determine the equilibrium allocations of human capital. This nonlinearity underlies Proposition 5.

household's consumption bundle is already distorted away from consumption of the cash good, and is therefore less sensitive to marginal increases in the inflation tax.

IV. A NUMERICAL EXAMPLE

In this section, a numerical example of the model economy is used to illustrate the basic propositions. The model is calibrated to annual data for the post-War US economy using the per capita gross growth rate of $\theta_{bm} = 1.015$ and gross inflation rate of $\pi_{bm} = 1.048$. Preference parameters of $\beta = 0.96$ and $a = 1$ (representing equal shares in consumption of the cash and non-cash goods)[17] and the factor share on human capital in the production of final goods of $\alpha = 0.64$ were chosen.[18]

Benchmark values for $(\delta h_y)_{bm}$ and R_{bm} can be found from (22) and (13).

$$(\delta h_y)_{bm} = [(\theta_{bm}/\beta) - 1]/(1-\alpha), \tag{27}$$

$$R_{bm} = [(\pi_{bm}\,\theta_{bm})/\beta] - 1. \tag{28}$$

Using the initial conditions for h, equation (16) can then be used to solve for the technology parameter, δ.

$$\delta = \{[(\delta h)_{bm}\,\Delta_{bm}] + (1-\psi_{bm})(1-\beta)\}/(\psi_{bm} h). \tag{29}$$

As an additional check on the reasonableness of this calibration, the long run interest semi-elasticity of money demand that is implied by the model is computed for a given velocity that approximates post-War US data.[19] For a benchmark velocity of 5·6, which corresponds to the average velocity of M1 over the period 1960:1–1992:2, the implied interest semi-elasticity of money demand is $-\cdot 004$. This number is slightly below (in absolute value) the range of $-0\cdot 08$ to $-0\cdot 13$ estimated by Stock and Watson (1993). This suggests that the welfare costs of inflation that are reported below are biased downward.

Under this calibration, the gross inflation rate is varied over the range $\pi \in [\beta, 1, \pi_{bm}, 1.1, 1.2, 1.3, 1.5, 1.75, 2, 3, 4]$, and equations (16)–(18) are used to determine the human capital allocations. Given these allocations, the gross growth rate under these alternative distortions could be found from equation (22). These results are reported in Table 1.

[17] The per capita growth figure came from Lucas (1990). The factor share for human capital is a typical choice for labour's share of income, as in Kydland and Prescott (1982). The inflation rate was calculated over the post-War period as the % change in the CPI. Initial conditions for h, k, and A were also required for the consumption calculations; however, none of the results reported in Table 1 were affected by alternative choices of initial conditions.

[18] The welfare calculations reported in Table 1 are very sensitive to the preference parameter a. Setting $a = 0.19$ [the parameter setting of Cooley and Hansen (1991)] which corresponds to a benchmark setting that requires 84% of consumption goods purchases to be cash-constrained, lowers the welfare losses for moderate inflations by approximately one-third. However, the losses remain high relative to what similar inflations would produce in a standard neoclassical growth model.

[19] Velocity, v, is measured as the sum of output from the final goods, intermediate goods, and transaction services sectors divided by real balances. This is given by: $v = (A_t h_y^{\alpha} x^{(1-\alpha)} + \eta A_t x + d_t z_t)/(M_t/p_t)$. In equilibrium, $v = (A_t h_y^{\alpha} x^{(1-\alpha)} + \eta A_t x + c_{2t})/c_{1t}$. Using benchmark values for v, h_y, and x, and the benchmark time paths for A_t, c_{1t}, and c_{2t}, a calibration for η can be found. With this calibration, velocity can be determined as a function of the distortion. The long run interest semi-elasticity in the vicinity of the benchmark inflation rate of 4·8% can then be computed as $[\log(v_{\pi=10\%}) - \log(v_{\pi=0\%})]/[R_{\pi=10\%} - R_{\pi=0\%}]$ with the units on the nominal interest rate in %.

Table 1

Effects of Inflation Taxes on the Allocation of Human Capital and Welfare

Parameter Settings: $\alpha = 0{\cdot}64$ (factor share on human capital),
$\beta = 0{\cdot}96$ (annual discount factor),
$a = 1$ (equal preferences for consumption goods).
Initial Conditions: A_0 (number of patents) = 100,
k_0 (stock of physical capital) = 100,
h (stock of human capital) = 100.
Benchmark Settings: $\theta = 1{\cdot}015$ (annualised per capita gross growth rate),
$\pi = 1{\cdot}048$ (annualised gross inflation rate).

		Shares of human capital in			Welfare losses	
Gross inflation rate, π	Gross growth rate, θ	Research sector, h_A	Payment system, h_z	Final goods, h_y	King-Rebelo measure, Λ_{kr}*	Cooley-Hansen measure, Λ_{ch}†
0·960	1·1059	6·125	31·598	62·276	−4·670	−4·462
1·000	1·0155	5·962	32.236	61·803	−2·540	−2·477
1·048	1·0150	5·778	32·951	61·271	0·000	0·000
1·100	1·0145	5·585	33·704	60.712	2·616	2·687
1·200	1·0136	5·243	35·034	59·723	7·434	8·031
1·300	1·0128	4·932	36·244	58·824	11·912	13·523
1·500	1·0114	4·384	38·378	57·234	19·878	24·810
1·750	1·0099	3·817	40·588	55·596	28·279	39·428
2·000	1·0087	3·343	42·433	54·224	35·230	54·392
3·000	1·0053	2·047	47·492	50·474	53·801	116·454
4·000	1·0033	1·273	50·490	48·236	64·333	180·368

* Λ_{kr} = percent reduction in period consumption needed to lower the lifetime utility level under the benchmark settings to the lifetime utility level under the alternative distortion. See footnote (21).

† Λ_{ch} = percent increase in period consumption under the alternative distortion needed to restore lifetime utility to its benchmark level. See footnote (21).

Note that as the inflation tax increases, more human capital is devoted to the payment system and less to research and final goods production, and growth is reduced. Under the benchmark setting, 5·78% of the stock of human capital is devoted to research, 33·95% to the payment system, and 61·27% to production. An increase in the net inflation rate of 15·2% above the benchmark (increasing π to 1·20) lowers the net per capita growth rate by 0·14% (reducing θ to 1·036), while the share of human capital devoted to the payment system increases 2·08% (to 35·054%) and the shares devoted to research and to production decline by 0·53% (to 5·24%) and by 1·55% (to 59·72%), respectively. At a 400% inflation rate, per capita growth falls to one-third of one percent, with over one-half of human capital being devoted to the payment system, and only 1·27% being devoted to research.

To perform the welfare calculations, it is first necessary to determine the initial levels of consumption, c_{10} and c_{20}.[20] Welfare calculations can then be

[20] From (2) and (22), the steady-state level of production of each intermediate good, x, is found, given the human capital allocation, h_y.

$$x = [(1-\alpha)A_0/(\delta_0 K_0)]^{[1/(\alpha-1)]}h_y.$$

Then, from equations (5), (8) and (12), with the rental rates for human capital in the final goods and transaction services sector set to their respective marginal products, the initial levels of consumption, c_{10} and c_{20}, can be determined.

made, following Cooley and Hansen (1989, 1991) and King and Rebelo (1990).[21] Since the two measures yield somewhat different quantitative results, both are reported in Table 1. In general, an increase in the inflation rate lowers welfare. Moreover, even seemingly small growth effects can have large welfare consequences. For example, note that a decline of the net inflation rate by 8·8% from the benchmark (to where $\pi = \beta$) corresponds to an increase in the net per capita growth rate by less than 0·1% (such that θ increases to 1·0159). However, this corresponds to an improvement in period consumption of approximately 4·5% ($\Lambda_{ch} = -4{\cdot}462\%$ and $\Lambda_{kr} = -4{\cdot}670\%$). These results are consistent with the findings of King and Rebelo (1990) and Lucas (1990) that distortions which affect growth rates, versus steady-state levels, as would be the case in a standard Solow-type neoclassical growth model, could have large welfare implications.

V. CONCLUSIONS

Most governments are monopoly providers of their economy's currency, and this currency usually establishes the economy's unit of account. These governments can therefore choose between alternative long run inflation policies by selecting the rate of monetary growth. Most have chosen high (relative to optimal) inflation policies that result in high nominal interest rates. This paper focuses on the social costs of inflation, when agents in the economy alter their means of payment in order to avoid the tax that inflation imposes on cash transactions. Effectively, the economy's real resources are unnecessarily channelled into the payment system, and the financial sector of the economy expands. This comes at the expense of output, and at the expense of activities that enhance growth, such as research. Consequently, consumption not only shifts to a new lower level as output falls, but the slower rate of economic growth induces cumulative losses over time. This may be of particular importance in economies, such as the one considered in this paper, where only second best policies are feasible.

The model economy of this paper is one where agents can purchase

$$c_{20} = \alpha x^{(1-\alpha)} h_t^{(\alpha-1)} h_z A_0,$$
$$c_{10} = [2/a(\theta\pi/\beta)] c_{20}.$$

The time paths for consumption, c_{1t} and c_{2t}, follow from the growth factor, θ.

[21] To obtain the measure of welfare losses induced by the distortion using Cooley and Hansen's approach, solve the following equation for *chx*.

$$\sum_{t=0}^{\infty} \beta^t U(\bar{c}_{1t}, \bar{c}_{2t}) = \sum_{t=0}^{\infty} \beta^t U[(1+chx)\,\hat{c}_{1t}, (1+chx)\,\hat{c}_{2t}],$$

where $\{\bar{c}_{1t}, \bar{c}_{2t}\}$ is the sequence of consumptions for the benchmark settings, and $\{\hat{c}_{1t}, \hat{c}_{2t}\}$ is the sequence of consumptions under the alternative distortions. Then, the welfare loss is measured as $\Lambda_{ch} = 100\%\ (chx)$, which is the percent increase in period consumption needed to restore lifetime utility to its benchmark level. An alternative method of calculating the welfare losses is given by King and Rebelo (1990) by solving the following equation for *krx*.

$$\sum_{t=0}^{\infty} \beta^t U[(1-krx)\,\bar{c}_{1t}, (1-krx)\,\bar{c}_{2t}] = \sum_{t=0}^{\infty} \beta^t U(\hat{c}_{1t}, \hat{c}_{2t}).$$

Here, $\Lambda_{kr} = 100\%\ (krx)$ represents the percent reduction in period consumption required to lower the benchmark lifetime utility to the level attained under the alternative distortion.

transaction services as an alternative to using cash. The transaction services technology requires human capital as the principal factor of production. However, human capital has alternative uses in the production of final goods, which include consumption from which households derive utility, and in the research sector, whose output is 'new knowledge'. This knowledge produces 'spillovers' into human capital activities and thereby raises productivity in the output, transaction services, and research sectors. The last of these generates growth endogenously. For this economy, it is shown that positive nominal interest rates induce welfare losses via level effects that distort human capital allocations away from final goods production, thereby lowering output, and that distort consumption choices away from cash constrained goods. Positive nominal interest rates also induce welfare losses via growth effects that distort human capital allocations away from research. This lowers the growth rate of output and consumption for all future periods, so its effects are cumulative. However, it was also found that the marginal effects of inflation taxes in generating these distortions are reduced as the inflation tax increases. At high inflation rates, the consumption bundle is comprised of relatively less of the cash constrained good, so the incidence of any incremental inflation tax is relatively smaller.

In the last section of the paper, a numerical example is given to illustrate the propositions associated with the theoretical model that is summarised above. It is demonstrated that, for reasonable parameter values, moderate inflations that induce seemingly small growth effects can produce substantial welfare losses. These losses occur solely through the misallocation of resources coming from changes in the usage of the economy's payment system.

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REFERENCES

Abel, A. B. (1985). 'Dynamic behavior of capital accumulation in a cash-in-advance model.' *Journal of Monetary Economics*, vol. 16, pp. 55–72.

Black, F. (1970). 'Banking and interest rates in a world without money.' *Journal of Banking Research*, vol. 1, pp. 9–20.

Coleman, W. J., II (1990). 'Money and capital in a cash-in-advance economy.' Unpublished manuscript, Board of Governors of the Federal Reserve System.

Cooley, T. and Hansen, G. (1989). 'The inflation tax in a real business cycle model.' *American Economic Review*, vol. 79, pp. 733–48.

Cooley, T. and Hansen, G. (1991). 'The welfare costs of moderate inflations.' *Journal of Money, Credit and Banking*, vol. 23, pp. 483–513.

De Gregorio, J. (1991). 'Inflation, taxation, and long run growth.' *Journal of Monetary Economics*, vol. 31, pp. 271–98.

De Gregorio, J. (1992). 'The effects of inflation on economic growth: lessons from Latin America.' *European Economic Review*, vol. 36, pp. 417–25.

Engineer, M. and Bernhardt, D. (1991). 'Money, barter, and optimality of legal restrictions.' *Journal of Political Economy*, vol. 99, pp. 743–77.

Friedman, M. (1969). *The Optimal Quantity of Money and Other Essays*. University of Chicago: Aldine.

Fuerst, T. (1992). 'Liquidity, loanable funds, and real activity.' *Journal of Monetary Economics*, vol. 29, pp. 3–24.

Ireland, P. N. (1994*a*). 'Endogenous financial innovation and the demand for money.' *Journal of Money, Credit and Banking*, (forthcoming).

Ireland, P. N. (1994*b*). 'Money and growth: an alternative approach.' *American Economic Review*, vol. 84, pp. 47–65.

Ireland, P. N. and Dotsey, M. (1993). 'On the welfare cost of inflation in general equilibrium.' Unpublished manuscript, Federal Reserve Bank of Richmond, August.

King, R. A. and Rebelo, S. (1990). 'Public policy and economic growth: developing neoclassical implications.' *Journal of Political Economy*, vol. 98, pp. S126–50.

Kydland, F. E. and Prescott, E. C. (1982). 'Time to build and aggregate fluctuations.' *Econometrica*, vol. 50, pp. 1345–70.

Lacker, J. M. and Schreft, S. L. (1991). 'Money, trade credit, and asset prices.' Unpublished manuscript, Federal Reserve Bank of Richmond.

Lucas, R. E. (1990). 'Supply-side economics: an analytical review.' *Oxford Economic Papers*, vol. 42, pp. 293–316.

Lucas, R. E. and Stokey, N. L. (1987). 'Money and interest in a cash-in-advance economy.' *Econometrica*, vol. 55, pp. 491–514.

Marquis, M. H. and Reffett, R. L. (1991). 'Real interest rates and endogenous growth in a monetary economy.' *Economic Letters*, vol. 37, pp. 105–9.

Marquis, M. H. and Reffett, R. L. (1992*a*). 'Anticipated inflation and capital in a multi-sector monetary economy.' *International Review of Economics and Finance*, vol. 2, pp. 129–47.

Marquis, M. H. and Reffett, R. L. (1992*b*). 'Capital in the payment system.' *Economica*, vol. 37, pp. 351–64.

Marquis, M. H. and Reffett, R. L. (1994). 'The inflation tax in a convex model of equilibrium growth.' *Economica*, (forthcoming).

Prescott, E. (1987). 'A multiple means of payment model.' In *New Approaches to Macroeconomics*, pp. 41–51.

Romer, P. M. (1990). 'Endogenous technological change.' *Journal of Political Economy*, vol. 98, pp. S71–102.

Schreft, S. L. (1992*a*). 'Transactions costs and the use of cash and credit.' *Economic Theory*, vol. 2, pp. 283–96.

Schreft, S. L. (1992*b*). 'Welfare-improving credit controls.' *Journal of Monetary Economics*, vol. 30, pp. 57–72.

Stock, J. and Watson M. W. (1993). 'A simple estimator of cointegrating vectors in higher-order integrated systems.' *Econometrica*, vol. 61, pp. 783–820.

Stockman, A. (1981). 'Anticipated inflation and the capital stock in a cash-in-advance economy.' *Journal of Monetary Economics*, vol. 8, pp. 387–93.

Townsend, R. (1987). 'Asset return anomalies in a monetary economy.' *Journal of Economic Theory*, vol. 41, pp. 219–47.